IPM Component 3 – STEM
Argumentation

Introduction

One way to think about an academic argument (noun: argumentation) is that it consists of two parts: i) is an assertion or statement that something is true or false, then ii) a reason as to why we believe that statement to be true or false. This reason must be logical and rational, and in science argumentation usually includes evidence. For example, the statement "dropping an object from a given height will cause it to fall to the ground because there is a force called gravity which acts on the object to attract it to the ground".

Note that argumentation and truth are two different things. For example,

"The sun and planets revolve around a stationary Earth
because when we look up at the sky the sun moves from East to West
and the planets move across the sky"

consists of an assertion and a reason why we believe this assertion is true. This assertion describes the geocentric system. But we now know that the Earth is not the centre of the planetary system, and that it (as well as all other planets) revolve around the Sun. So we can believe in something, and provide a proper argument for that belief, *but still be wrong in our belief*.

As another example consider Newton's theory of gravity which predicted that Uranus should be in a certain position at a certain time in its orbit around the sun. However, detailed measurements during the mid 1800s showed that this was not the case. Hence we have

- *Assertion:* "There is a planet beyond Uranus"
- *Reason:* "The orbit of Uranus is perturbed, so there is a new planet beyond Uranus which has a gravitational effect on the latter and explains why Uranus is not where we believe it should to be"
- *Confirmation:* "Telescopic observation is conducted, and the planet is found in the location predicted by Newton's theory of gravity."

The planet was named Neptune.

As another example consider that it is an accepted fact that water boils at 100^0C. Then

- *Assertion:* "Water can boil at higher or lower temperatures"
- *Reason:* "The boiling point of water depends on pressure and any substances added. Higher altitudes lead to lower boiling points of water, and different substance lead to higher boiling points of water."
- *Confirmation:* "Conduct an experiment at 1900 metres to find that water boils at ~93^0C."

In the examples of the discovery of Neptune and different boiling points of water the argumentation consists of the actual experiments conducted to confirm the hypothesis. The analysis and conclusion of experimental results is a form of argumentation.

As a more detailed example of consider the case of Phlogiston. This was an early chemical theory relating to combustion. Phlogiston was supposed to be a substance inherent in matter which was released during combustion. What was left upon dephlogistication was an ash or residue. The main reason for believing in phlogiston was that when substances burn the residue or ash is lighter than the original substances, so that something must have been released during the process of combustion.

As such, wood was considered a combination of phlogiston and wood ash. Another example was the corrosion of metals in air (e.g., the rusting of iron). This was considered to be a form of combustion, so that when a metal was converted to its metallic ash (its oxide, in modern terms), phlogiston was lost. Therefore, metals were composed of metallic ash and phlogiston.

In general, substances that burned in air were said to be rich in phlogiston. The fact that combustion soon ceased in an enclosed space was taken as evidence that air had the capacity to absorb only a finite amount of phlogiston. When air had become completely phlogisticated (in other words, saturated) it would no longer serve to support combustion of any material, nor would a metal heated in it yield a calx.

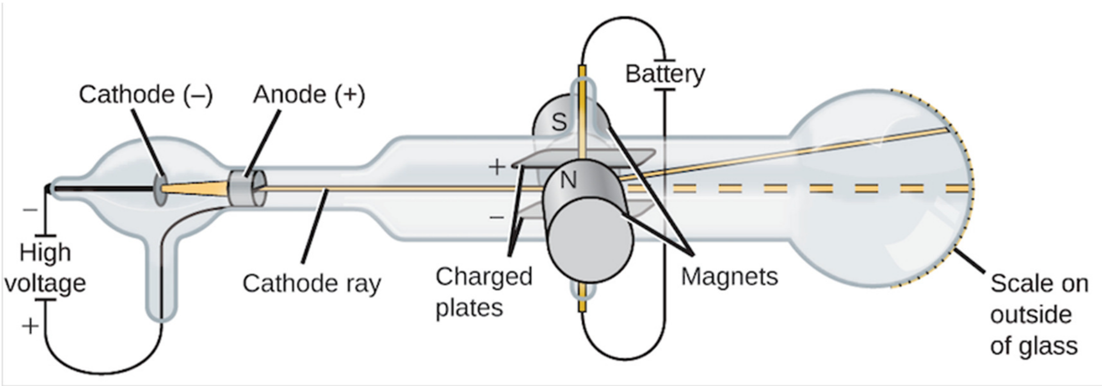
As chemistry advanced, phlogiston was considered a true substance, and much effort was expended in accounting for the weight changes observed. When hydrogen, very light in weight and extremely flammable, was discovered, some thought it was pure phlogiston. The major objection to the theory, that the ash of organic substances weighed less than the original while the calx was heavier than the metal, was brushed aside by certain scientists of the 1700s.

The phlogiston theory was discredited by Antoine Lavoisier between 1770 and 1790. He studied the gain or loss of weight when tin, lead, phosphorus, and sulphur underwent reactions of oxidation or reduction (de-oxidation); and he showed that the newly discovered element oxygen was always involved. Despite these consistent results involving oxygen, a number of chemists tried to retain some form of the phlogiston theory. one such was Joseph Priestly (who actual discovered oxygen). Oxygen is now known to strongly support combustion, and can allow substances to burn for longer, but Priestly believed oxygen to be dephlogisticated air capable of combining with more phlogiston and thus supporting combustion for longer than ordinary air. However, by 1800 practically every chemist recognized the correctness of Lavoisier's oxygen theory.

Hence

- *Assertion:* "All matter contains a substance which we call phlogiston"
- *Reason:* "The residue/ash produced after burning some substances is lighter than the original substance.
- *Confirmation:* "Systematic experiments studying the gains or losses of weight in a number of substances, where every gain or loss involved this new element called oxygen."
- *Validity:* Phlogiston was successful at explaining certain aspects of combustion. but it ultimately proved incorrect. Matter does not contain any substance such as phlogiston. The real process of combustion involves oxygen, which was discovered as part of the process of refuting the theory of phlogiston.

Another example comes from physics. Before the 19^{th} century the atom was thought to be an indivisible particle. But by the 19th century this was starting to be doubted. Then J. J. Thomson conducted an experiment to test the assertion that there existed a negatively charged particle. Using equipment similar to that shown below he found, in 1897, that a beam of particles projected from the cathode (left hand end of the diagram below) was deflected away from the negatively charged plate (middle of the diagram) to strike the top part of the glass bulb (right hand side of the diagram).



Hence

- *Assertion:* "There exists a negatively charged particle inside the atom."
- *Reason:* "Any beam of particles which are negatively charged will be deflected away from positive charges."
- *Confirmation:* "Experiments using a cathode ray tube, containing positively and negatively charged plates, deflect the cathode beam away from the positively charged plate. The cathode ray strikes the glass bulb not in the centre but towards the top, in a direction away from the negatively charged plate."

See the following link for more: https://www.khanacademy.org/science/chemistry/electronic-structure-of-atoms/history-of-atomic-structure/a/discovery-of-the-electron-and-nucleus

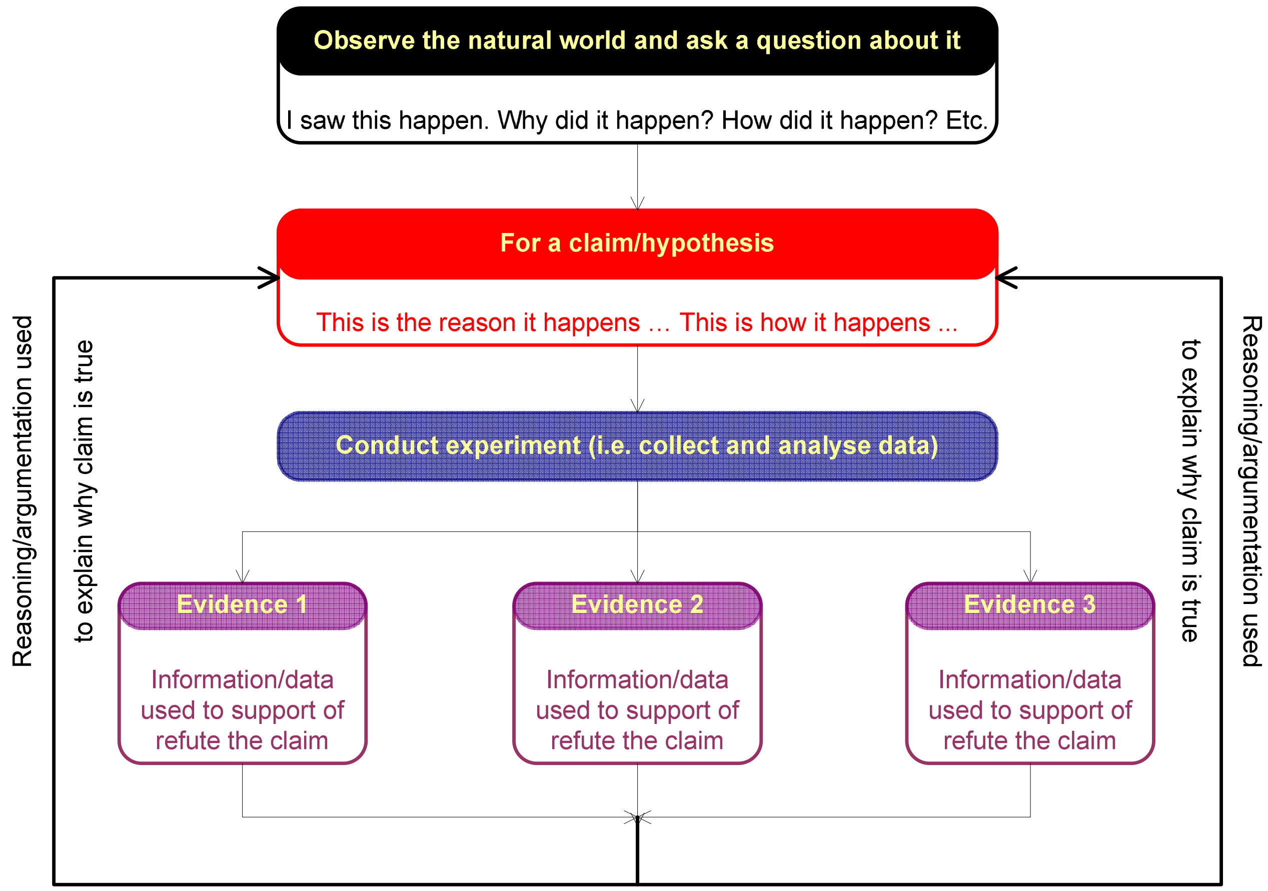
Another way to think of argumentation is as a form of *justification* or *explanation* of what was done, how it was done, why it was done, etc… For example,

- Statistics: i) "There is a statistical difference between the means of two random samples. This is shown by the results of a *t*-test using a *p* value of 0.01."; ii) "The value of *g*, the acceleration due to gravity, is approximately 9.8 m/s^2. This is based on having performed a 95% confidence-interval test on experimental data"; iii) A box contains 1000 rubber bands whose average breaking strength is known to be 4.5kg. A random sample of 50 rubber bands taken from the box containing was found to have a breaking strength of 4.35 kg. Statistical tests show that this breaking strength not usual since it lies beyond the accepted range of breaking strengths."
- Mechanics: i) "In order to simplify the problem we consider that the object as a particle"; ii) "Since friction is negligible we will ignore it, and treat the surface of a table

as smooth"; iii) "The object is one-thousand times heavier than the rod. As a result, we treat the rod as being without weight".

- Mathematics: "Points of inflections of a function can sometimes be found via the first derivative. An example of this can be shown for $y = x^3$. In this case we differentiate the function, equate it to 0 and solve for x. We then need to test this value of x to confirm that it is a point of inflection. However, not all points of inflections of a function can be found via the first derivative. Some points of inflection need to be found via the second derivative. An example of this is the point of inflection associated with $y = x^2(x-2)$. We therefore, differentiate the first derivative, solve for x in this second derivative and then test the resulting values of x to confirm (or not) any points of inflection."

The diagram below shows where argumentation is located within the context of research. Do you agree with this?



Recall the initial example of "dropping an object from a given height will cause it to fall to the ground because there is a force called gravity which acts on the object to attract it to the ground". The assertion can be made more specific, such as

i) "dropping an object from a height up to a given maximum height will cause it to fall to the ground", since if we go high enough (i.e. into space) the object will not fall back to the ground;

ii) "dropping an object from a height up to a maximum height will cause it to fall to the Earth at a certain speed", since different planets have a different force of gravity (for example, the moon's gravity is about 18% as strong as the Earth gravity)

In both cases argumentation, based on experimental data, will need to be developed in order to justify such assertions.

Exercise

1) What kind of argumentation do you thing would be needed to justify assertions i) and ii) above?

2) What assertions, claims or theories were used in your discipline in the past that are now no longer used? What arguments were provided to support these claims? Why are these claims no longer accepted?

We now look at two other forms of argumentation: i) "A did X because of Y", and ii) "Since/If – Then – Because"

Example 1: "A did X because of Y"

The following text is taken from https://plato.stanford.edu/entries/statistics/#NatEvi which is Stanford University' encyclopaedia on the philosophy of statistics.

> "Imagine two researchers who are both testing the same lady on her ability to determine the order in which milk and tea were poured in her cup. They both entertain the null hypothesis that she is guessing at random, with a probability of 1/2, against the alternative of her guessing correctly with a probability of 3/4. The more diligent researcher of the two decides to record six trials. The more impatient […] researcher records at most six trials, but decides to stop recording the first trial that the lady guesses incorrectly. Now imagine that, in actual fact, the lady guesses all but the last of the cups correctly. Both researchers then have the exact same data of five successes and one failure, and the likelihoods for these data are the same for the two researchers too. However, while the diligent researcher cannot reject the null hypothesis, the impatient researcher can."

Using the technique of "A did X because of Y" we can trace the development of the argument above:

> "Imagine two researchers **[A1 and A2]** who are both testing **[X]** the same lady on her ability to determine the order in which milk and tea were poured in her cup **[Y].** They both entertain the null hypothesis that she is guessing at random, with a probability of 1/2, against the alternative of her guessing correctly with a probability of 3/4.
>
> The more diligent researcher **[A1]** of the two decides to record six trials **[X][Y]**. The more impatient […] researcher **[A2]** records at most six trials **[X][Y]**, but decides to stop recording **[X]** the first trial that the lady guesses incorrectly **[Y]**.
>
> Now imagine that, in actual fact, the lady guesses all but the last of the cups correctly. Both researchers **[A1 and A2]** then have the exact same data of five successes and one failure, and the likelihoods for these data are the same for the two researchers too. However, while the diligent researcher **[A1]** cannot reject **[X]** the null hypothesis, the impatient researcher **[A2]** can **[X]**."

The previous analysis used the researchers to be A. What if we analyse the text with the lady being A? This is what we obtain:

> "Imagine two researchers who are both testing the same lady **[A]** on her ability to determine the order in which milk and tea were poured **[X]** in her cup. They both entertain the null hypothesis that she is guessing **[Y]** at random, with a probability of 1/2, against the alternative of her guessing correctly with a probability of 3/4.
>
> The more diligent researcher of the two decides to record six trials. The more impatient […] researcher records at most six trials, but decides to stop recording the first trial that the lady guesses incorrectly.
>
> Now imagine that, in actual fact, the lady **[A]** guesses **[X]** all but the last of the cups correctly. Both researchers then have the exact same data of five successes and one failure, and the likelihoods for these data are the same for the two researchers too. However, while the diligent researcher cannot reject the null hypothesis, the impatient researcher can."

When using the approach of "A did X because of Y" which A do you think is most relevant to use? The researchers or the lady?

Example 2: "Since/If – Then – Because"

The following text is taken from https://plato.stanford.edu/entries/statistics/#NatEvi which is Stanford University' encyclopaedia on the philosophy of statistics.

> "Statistics investigates and develops specific methods for evaluating hypotheses in the light of empirical facts. A method is called statistical, and thus the subject of study in statistics, if it relates facts and hypotheses of a particular kind: the empirical facts must be codified and structured into data sets, and the hypotheses must be formulated in terms of probability distributions over possible data sets. The philosophy of statistics concerns the foundations and the proper interpretation of statistical methods, their input, and their results. Since statistics is relied upon in almost all empirical scientific research, serving to support and communicate scientific findings, the philosophy of statistics is of key importance to the philosophy of science. It has an impact on the philosophical appraisal of scientific method, and on the debate over the epistemic and ontological status of scientific theory."

Using the technique of "Since/If – Then – Because" we can trace the development of the argument above. In the text below I have reorganised certain parts in order to emphasise the "Since/If – Then – Because" aspect. See if you can re-read the text above with this in mind.

> "Statistics investigates and develops specific methods for evaluating hypotheses in the light of empirical facts. **IF** the empirical facts [can] be codified and structured into data sets, and **IF** the hypotheses [can] be formulated in terms of probability distributions over possible data sets **THEN** a method used on this data can be called statistical **BECAUSE** it relates facts and hypotheses of a particular kind
> [...]
> The philosophy of statistics concerns the foundations and the proper interpretation of statistical methods, their input, and their results. **SINCE** statistics is relied upon in almost all empirical scientific research, serving to support and communicate scientific findings, **THEN** the philosophy of statistics is of key importance to the philosophy of science, **BECAUSE** statistics is relied upon in almost all empirical scientific research, it has an impact on the philosophical appraisal of scientific method, and on the debate over the epistemic and ontological status of scientific theory."

Other examples

A mathematics text

Consider the texts below.

> "Many workshops and meetings with the US high school mathematics teachers revealed a lack of familiarity with the use of transformations in solving equations and problems related to the roots of polynomials. When asked to find a quadratic equation whose solutions are reciprocals of $ax^2 + bx + c = 0$, the teachers uniformly tried to answer the question using the quadratic formula and could not generalize the problem and the answer to n^{th} degree equations.
>
> We introduced the use of the substitution $x = 1/y$ as a way of solving this problem, with the intention that it would help the teachers learn to generalise not only the reciprocation of roots but also give the ideas about how to transform roots in different ways. As a result of demonstrating the use of the substitution teachers were able to find an equation whose solutions are twice (or n times) as large as the solutions of a given equation, or increased by a constant.
>
> The workshop participants were also introduced to the two approaches for deriving the quadratic formula described in this article. They believed that their students will benefit from the transformational approach."

Libeskind, Shlomo (2010) "The use of transformations in solving equations", *International Journal of Mathematical Education in Science and Technology*, **41**(3), p432 - 434

Below is an analysis according to both the "Since/If-Then-Because" and "A did X because of Y to get Z" form of argumentation.

[SINCE teachers have a lack of familiarity with ...] [Because of Y]	"Many workshops and meetings with the US high school mathematics teachers revealed a lack of familiarity with the use of transformations in solving equations and problems related to the roots of polynomials.

	When asked to find a quadratic equation whose solutions are reciprocals of $ax^2 + bx + c = 0$, the teachers uniformly tried to answer the question using the quadratic formula and could not generalize the problem and the answer to n^{th} degree equations.
[THEN…] [A did X to get Z]	We introduced the use of the substitution $x = 1/y$ as a way of solving this problem,
[BECAUSE…] [to get Z (more detailed)]	with the intention that it would help the teachers learn to generalise not only the reciprocation of roots but also give the ideas about how to transform roots in different ways.
[CONSEQUENCE / OUTCOME…]	As a result of demonstrating the use of the substitution teachers were able to find an equation whose solutions are twice (or n times) as large as the solutions of a given equation, or increased by a constant.

A statistics text

Consider the texts below. Even if you do not understand the description about the Chis-squared test it should be possible for you to identify, by the use of the language used in the text, where argumentation occurs. Therefore, analyse the text according to both the "Since/If-Then-Because" and "A did X because of Y to get Z" forms.

> The chi-squared test is a test which allows for variables to be grouped together in order to understand the correlation between the different variables. By showing how correlations change from one group of variables to another, the chi-squared test allows for the identification of patterns, trends, and probabilities within data sets.
>
> The Chi-Square test is most useful when analyzing contingency tables of survey response data. Because contingency tables reveal the frequency and percentage of responses to questions by various segments or categories of respondents (gender, profession, education level, etc.), the Chi-Square test informs researchers about whether or not there is a statistically significant difference between how the various segments or categories answered a given question.

> However, chi-square only tests whether two individual variables are independent in a binary, "yes" or "no" format. Chi-square testing does not provide any insight into the degree of difference between the respondent categories, meaning that researchers are not able to tell which statistic (result of the Chi-Square test) is greater or less than the other. Secondly, chi-square requires researchers to use numerical values, also known as frequency counts, instead of using percentages or ratios. This can limit the flexibility that researchers have in terms of the processes that they use.

A mathematical modelling text 1

Consider the texts below. Again, even if you do not understand the description about the Chis-squared test it should be possible for you to identify, by the use of the language used in the text, where argumentation occurs. Therefore, analyse the text according to both the "Since/If-Then-Because" and "A did X because of Y to get Z" forms.

> "While educational research on mathematical modelling is extensive, not much attention has been paid to empirical investigations of how modelling as practiced in industry can be adapted to school settings. The fact that mathematical modelling has been taught in different ways in different national curricula (e.g., Ärlebäck, 2009; Blomheij & Hoff Kjeldsen, 2006; Schmidt, 2012), indicates the problematic nature of teaching industry practice in schools. Dierdorp, Bakker, van Maanen, and Eijkelhof (2014) argue that students find education more meaningful and useful when it draws on "problems in authentic professional practices" (p. 3). Empirical research into the process of "educationalising" the industry practice of mathematical modelling within a school setting would provide knowledge for developing teaching and learning approaches for students in their future professional activities as constructors and users of models (cf. Drakes, 2012; Gainsburg, 2003)."

(text adapted from "Mathematical modelling as a professional task", Peter Frejd and Christer Bergsten, *Educational Studies in Mathematics*, January 2016, Vol. 91, No. 1 (January 2016), pp. 11-35.

A mathematical modelling text 2

Consider the texts below. Again, even if you do not understand the description about the Chis-squared test it should be possible for you to identify, by the use of the language used in the text, where argumentation occurs. Therefore, analyse the text according to both the "Since/If-Then-Because" and "A did X because of Y to get Z" forms.

> "Recent work on validation has demonstrated the paucity of validation in operational research projects. Gass' shows that much of validation is only simple verification, whilst work by Finlay and Wilson[2] showed that little validation was carried out on 'in-house' work, but that a little more was included when outside consultants were called in by an organization.
>
> This paper will consider the processes of mathematical model-building and validation as a function of the level of contact between the modeller and the decision-maker. It is the contention of the authors that different levels of contact require emphasis on different types of validation. Examples, mainly from case studies involving linear programming, will be used to reinforce these statements. The extent to which linear programming is still important for US business applications is discussed in Kathawala,[3] and the way in which it is used in several countries is discussed in Lockett.' Linear programming is also a good area to consider because it is probably the area in operational research practice where the separation between modeller and decision-maker is at its greatest."

(From "Orders of Validation in Mathematical Modelling" P. N. Finlay and J. M. Wilson, *The Journal of the Operational Research Society* , Feb., 1990, Vol. 41, No. 2 (Feb., 1990), pp. 103-109.

Comparing argumentation with description

Last week we saw the difference between a critique and a summary, whereby a summary could be seen simply as a description, and a critique could be seen as a justification. Argumentation is different from critique and a summary in that argumentation involves a particular type of comparison, contrast and justification. For example

SINCE <*something has happened differently than expected or has not happened at all*>

THEN <*you can justify why you want to change things*>

BECAUSE <*you want to can obtain better results or see how things behave differently*>

This is summarised in the table below.

Descriptive writing	**Argumentation writing**
States what happened.	Describes what else could have happened under different conditions or circumstances.
States what something is like.	Describes how things might have been different in different circumstances; Compares this with something else and justifies why you are changing it.
State the order in which things happen.	Compare these things and justify why some are better/worse than the others, or why the order could have been different.
Explain what a theory says.	Compares the theory with other theories and justifies why it is better or worse; compares assumptions and flaws between theories and justifies their merits or not.
States and describes the methods or methodology used	Compares methods and methodologies in order to indentify the best one to use and justifies why this is the best one to use.
Gives information.	Compares information in order to draw conclusion.

The language and discourse of argumentation

The examples above on argumentation involved certain types of vocabulary, phrasing and sentence building. The way in which this vocabulary and phrasing can be built is iluustrated in the table on the next page.

The aim of this table is to show you examples of an *underlying principle* of what constitutes *argumentation language*. This underlying principle is what you should aim to learn and understand. Then you will know *how* to write an argument, and you will only need to learn individual vocabulary, terminology, and phrasing in order to write arguments in your own discipline.

For example, given the following text

- The *t*-test was used as a way of determining whether or not there was a statistically significant difference between the means of two samples. We were able to use this test because the two samples came from populations which were (approximately) normally distributed. If they had not been so distributed then the results would have not been valid. Furthermore, the samples were drawn at random in order to minimise bias when collecting the data. Had this not been done the comparison of means would have been invalid.

we can rewrite this as illustrated below whilst retaining the features and essence of an argument

- Given two populations which were (approximately) normally distributed it was possible to collect a random sample of data from each population. Normal distributed data was necessary in order to make any test valid, and random sampling was necessary in order to minimise bias. To have not chosen the samples randomly would render any comparison of means invalid. The way in which the samples means were compared was by using a *t*-test in order to determine if there was a statistically significant difference between them.

Because X is	is similar to is different from can be contrasted with ...	Y in	several a number of ...	ways means respects ...	we	introduce alter add assume ...
Due to the	similarites differences changes deviatiions ...	in ... from ... with respect to ...	as a result of due to because of by reason of in order to account for ...	we have		introduced altered added assumed ...
X is/has a	important significant major ...	impact determinant tendency trend ...	on/towards ... as a result of	alterations additions modifications changes adjustments ...		made to ...

Note that the examples above are not exhaustive. It is only by doing a lot of reading that you come to know what argumentation looks and how they are written. This leads us to the fact that argumentation is seen to be argumentation by the use of a particular type of vocabulary constructed via phrasing and sentences and paragraphs in a particular way.

So we might say that

argumentation is about using language which illustrate
one's close, reflective reading of the text,
where such language is seen via aspects of comparing and contrasting,
discussing advantages and disadvantages, pros and cons, cause and effect, etc.

Areas where argumentation can be found

The list below (which is not exhaustive) illustrate where argumentation can be found:

1. diagnosing problems,
2. discussing the validity of assumption,
3. justifying actions and decisions,
4. comparing and contrasting two or more things (for example, advantages and disadvantages),
5. describing changes (in behaviour of over time, etc.), or development (i.e. from old theories and methods to new theories and methods),
6. investigating something in depth, such as systematically examining the details of data collection, metholody, analysis, conclusions, etc.
7. suggesting improvements or changes to existing work,

etc.

Exercise

Each of the texts below illustrate a form of argumentation.

1. In each text there is one sentence which is irrelevant because it does not fit in with the argument being made. Identify this sentence and explain why it does not fit.
2. Each text is arguing in favour of a particular idea. What is that idea?

1) **(I)** Science is systematic because of the attention it gives to organising knowledge and making it readily accessible to all who wish to build on its foundation. **(II)** If the results support the hypothesis, the scientist may use them to generate related hypotheses. **(III)** In this way science is both a personal and social endeavour. **(IV)** In other words, it is beneficial to both the individual and to society at large. **(V)** Therefore, science contributes a great deal to the improvement and the quality of life.

2) **(I)** As people age, the amount of water in the body decreases. **(II)** Since many drugs dissolve in water and since less water is available in the body to dilute them, these drugs reach higher levels of concentration in the elderly. **(III)** Also, the kidneys are less able to excrete drugs into the urine, and the liver is less able to metaboise many drugs. **(IV)** For these reasons, many drugs tend to stay in an elderly person's body much longer than in a young person's body. **(V)** People in every civilisation in recorded history have used drugs of plant and animal origin to prevent and treat disease.

3) **(I)** Seismic waves are the vibratoins from earthquakes that travel through the Earth. **(II)** The Richter scale was developed in 1935 by Charles F. Richter of the California Institute of Technology to compare intensity of earthquakes. **(III)** They are recorded on instruments called seismographs. **(IV)** Seismographs record a zigzag trace that shows the varying amplitude of ground oscillations beneath the instrument. **(V)** Sensitive seismographs, which greatly magnify these ground movements, can detect strong earthquakes from sources anywhere in the world.